جزوه هندسه اصلی

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s13200670r@gmail.com

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خطوط موازی و مورب (زاویه ها)

Chapter 1 Geometry

Parallel lines intersected by a transversal

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۱.اصل خطوط موازی و مورب

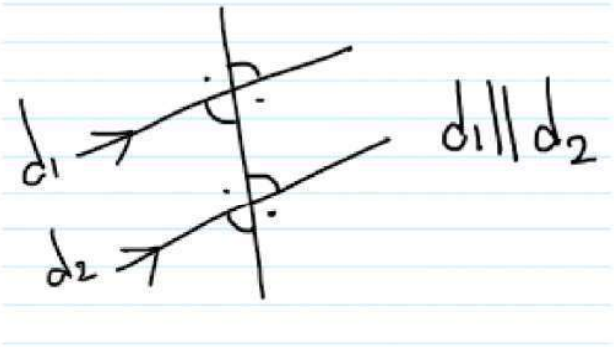
زوایای متقابل به راس

جمع دو زاویه کنار هم = ۱۸۰

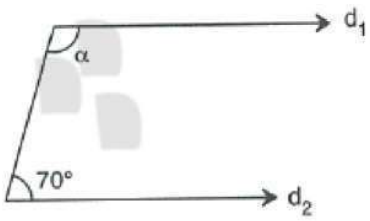
جمع زاویه یک دور کامل = ۳۶۰

معرفی نمادهای ریاضی

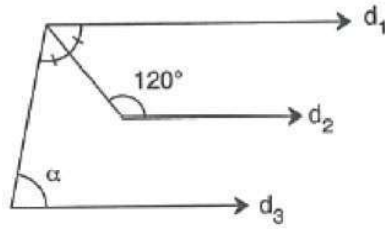
عمود، موازی، زاویه، پاره خط و نیم خط



مثال : ساختار U

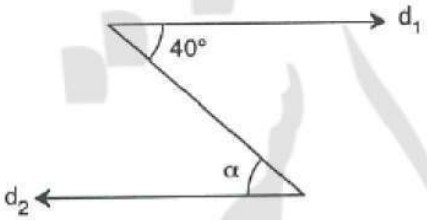


$d_1 // d_2$
$\Rightarrow \alpha = ?$

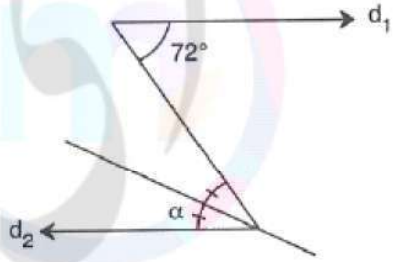


$d_1 // d_2 // d_3$
$\Rightarrow \alpha = ?$

مثال :ساختار Z

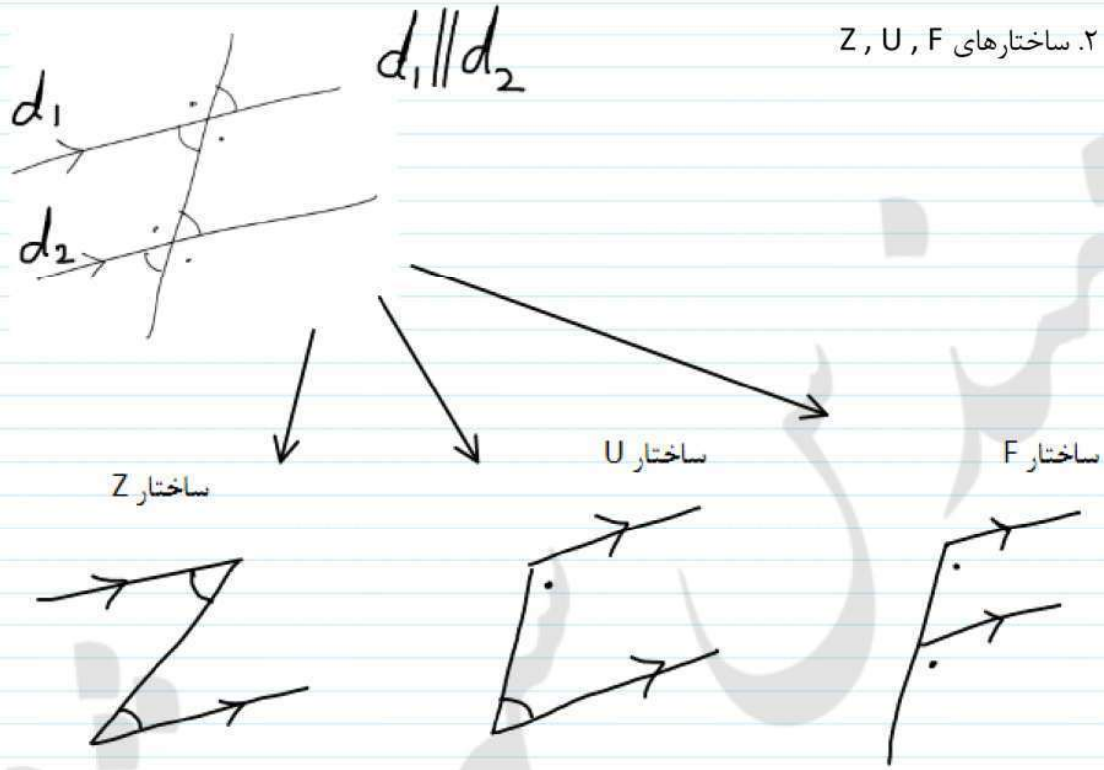


$d_1 // d_2$
$\Rightarrow \alpha = ?$

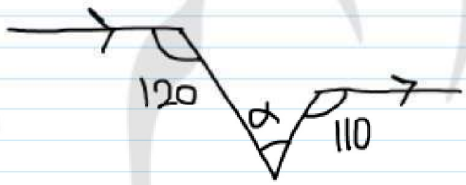


$d_1 // d_2$
$\Rightarrow \alpha = ?$

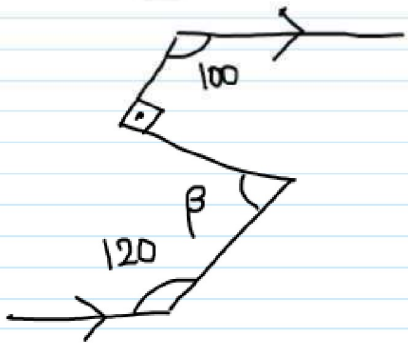
۲. ساختارهای Z , U , F



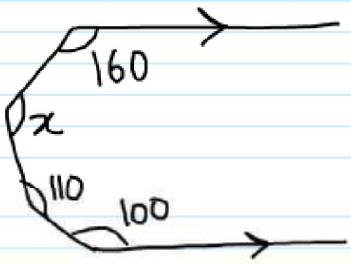
مثال: $\alpha = ?$



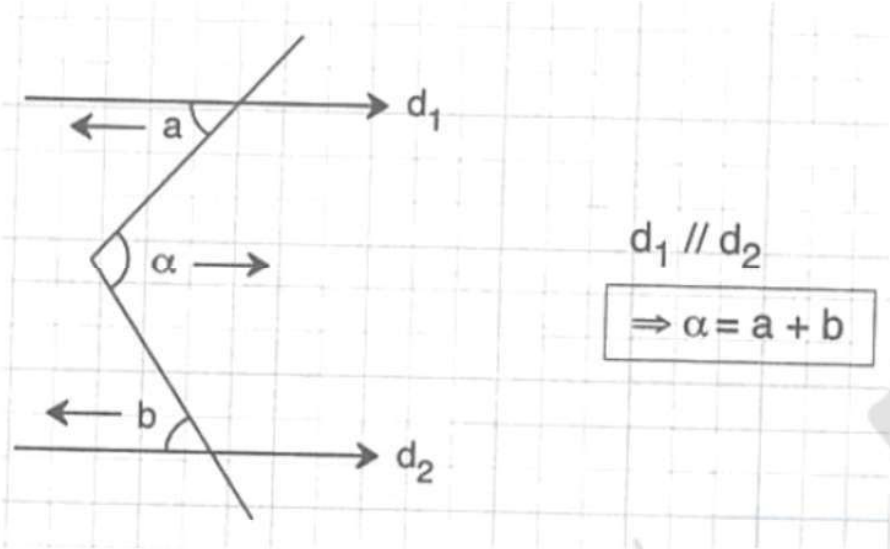
مثال: $\beta = ?$



مثال: $x = ?$



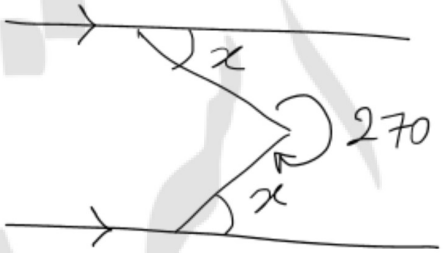
۳. ساختار Σ



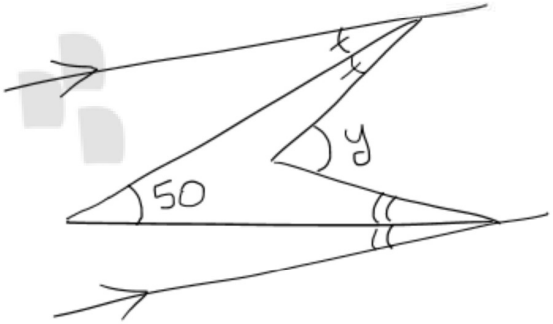
$d_1 // d_2$

$\boxed{\Rightarrow \alpha = a + b}$

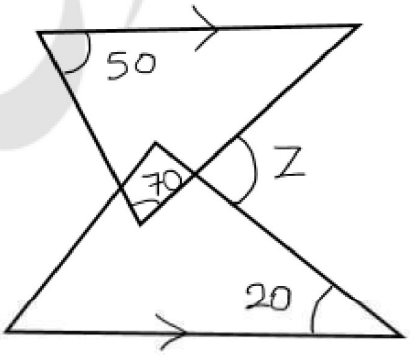
مثال : $x = ?$



مثال : $y = ?$



مثال : $Z = ?$

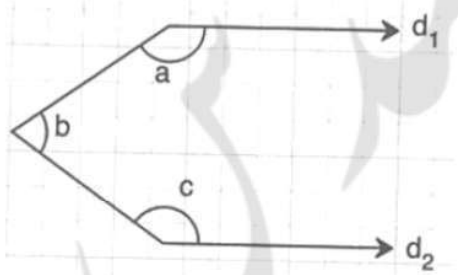


۴. ساختار Σ و U تعمیم یافته

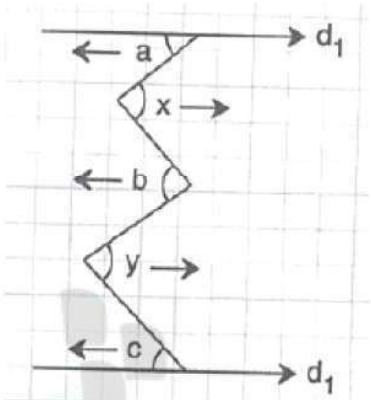
ساختارهای U تعمیم یافته

ساختار Σ تعمیم یافته

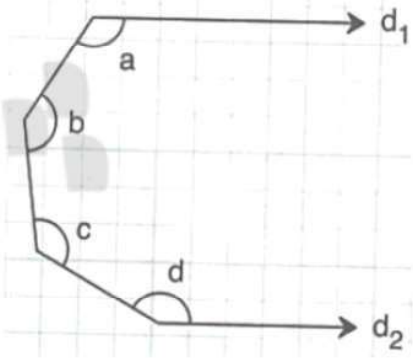
$a+b+c = 2x180 = 360$

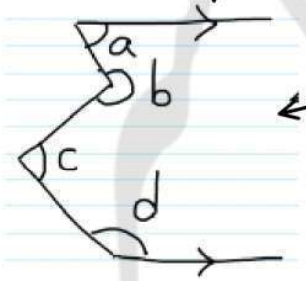


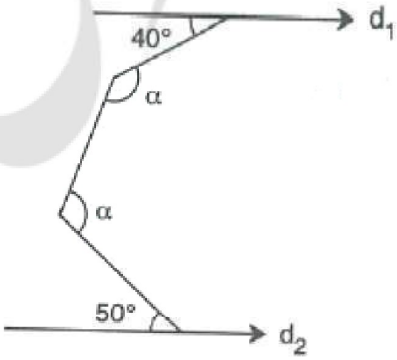
$x+y = a+b+c$



$a+b+c+d = 3x180 = 540$





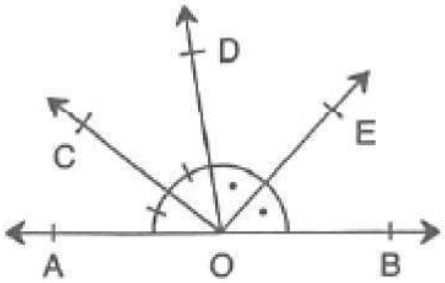


$d_1 // d_2$

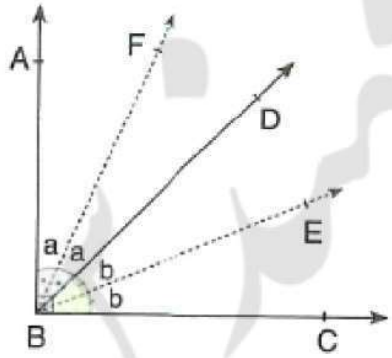
$\Rightarrow \alpha = ?$

۵. ساختارهای خاص ۱

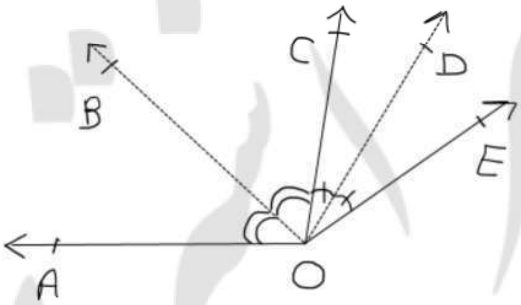
$C\hat{O}E = ?$



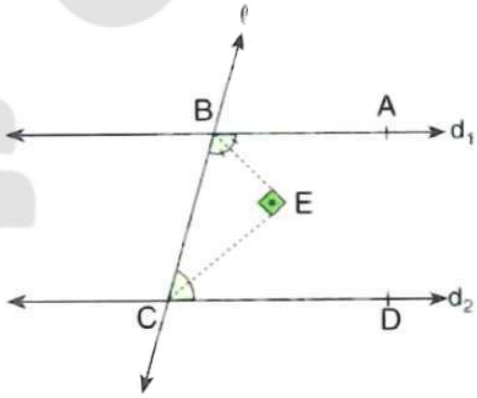
$F\hat{B}E = ?$



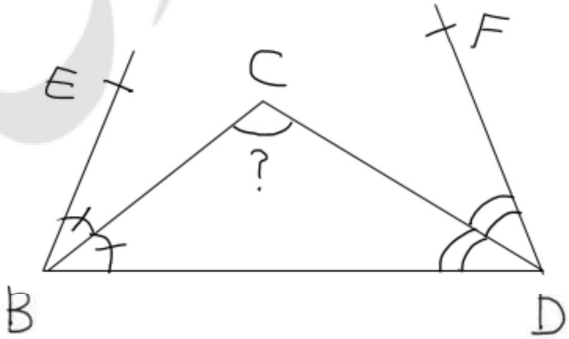
$A\hat{O}E = 120 \Rightarrow B\hat{O}D = ?$



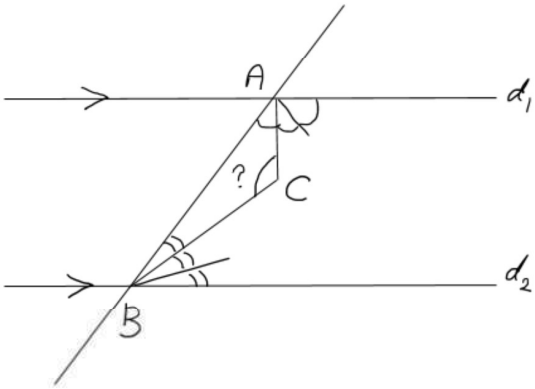
$B\hat{E}C = ?$



$D\hat{B}E + B\hat{D}F = 140 \Rightarrow B\hat{C}D = ?$

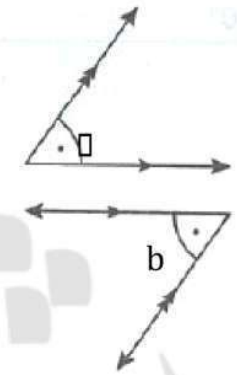


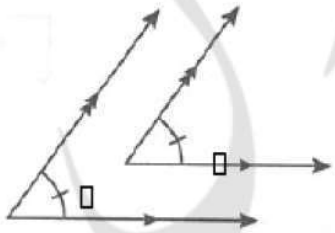
$B\hat{C}A = ?$

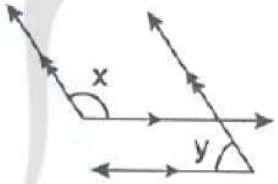


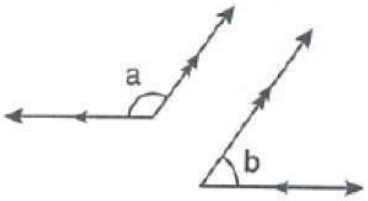
۶. جفت خط های موازی

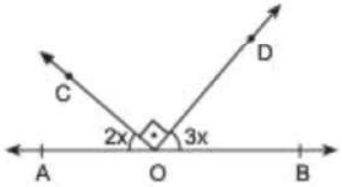
درک موازی مورب!









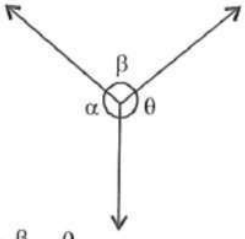


Şekilde, $m(\widehat{COD}) = 90°$, $m(\widehat{AOC}) = 2x$, $m(\widehat{DOB}) = 3x$

olduğuna göre, x açısının ölçüsü kaç derecedir?

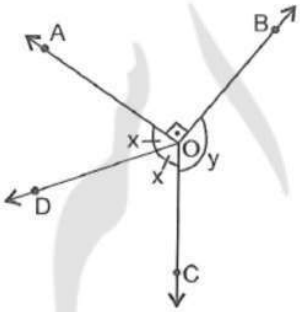
A) 15 B) 16 C) 17 D) 18 E) 20

تمرینات جبری



$\alpha, \beta, \theta, \frac{\alpha}{2} = \frac{\beta}{3} = \frac{\theta}{5} \Rightarrow \alpha = ?$

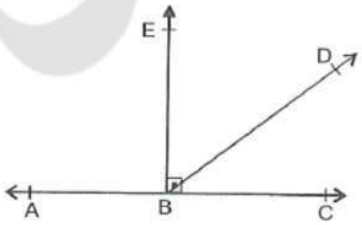
A) 60 B) 72 C) 80 D) 96 E) 108



$[OA \perp [OB$, $m(A\hat{O}D) = m(C\hat{O}D) = x$ $m(B\hat{O}C) = y$

$y - x = 15° \Rightarrow$ **x =?**

A) 65 B) 70 C) 75 D) 80 E) 85



$m(A\hat{B}D) = 5m(C\hat{B}D)$

$\Rightarrow m(E\hat{B}D) = ?$

A) 30 B) 45 C) 50 D) 54 E) 75

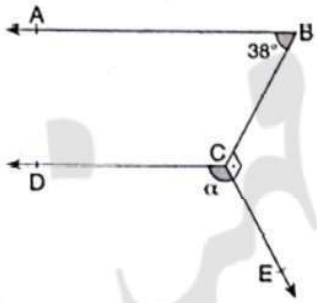
تمرین

$[BA // [CD$

$m(\widehat{ABC}) = 38°$

$m(\widehat{BCE}) = 90°$

$\Rightarrow m(\widehat{DCE}) = \alpha = ?$



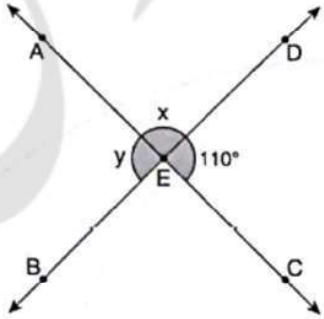
A) 128° B) 130° C) 131° D) 132° E) 133°

$m(\widehat{DEC}) = 110°$

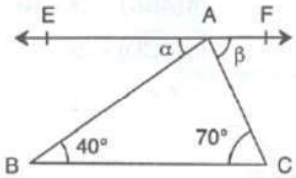
$m(\widehat{AED}) = x$

$m(\widehat{AEB}) = y$

$\Rightarrow y - x = ?$



A) 30° B) 35° C) 40° D) 45° E) 50°



$[EF] // [BC]$

$m(\widehat{EAB}) = \alpha$

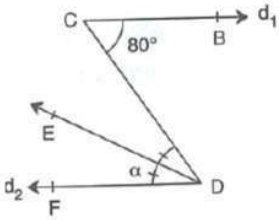
$m(\widehat{ABC}) = 40°$

$m(\widehat{FAC}) = \beta$

$m(\widehat{ACB}) = 70°$

$\Rightarrow \beta - \alpha = ?$

A) 30 B) 40 C) 50 D) 60 E) 70



$d_1 // d_2$

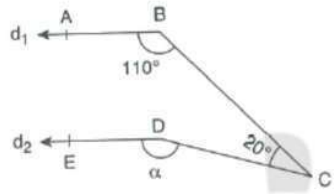
$m(\widehat{BCD}) = 80°$

$\Rightarrow m(\widehat{CDE}) = m(\widehat{EDF}) = \alpha = ?$

A) 5 B) 10 C) 40 D) 50 E) 100

تمرین

سبک ۱



$d_1 // d_2$

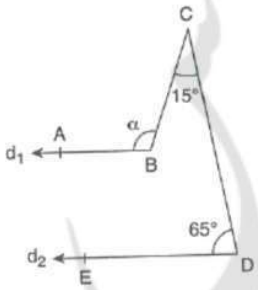
$m(\widehat{ABC}) = 110°$

$m(\widehat{BCD}) = 20°$

$m(\widehat{EDC}) = \alpha$

$\Rightarrow \alpha = ?$

A) 50 B) 70 C) 80 D) 110 E) 130



$d_1 // d_2$

$m(\widehat{BCD}) = 15°$

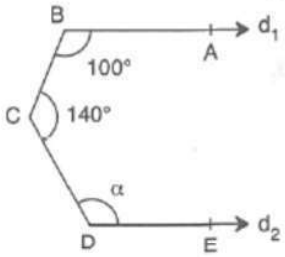
$m(\widehat{EDC}) = 65°$

$m(\widehat{ABC}) = \alpha$

$\Rightarrow \alpha = ?$

A) 70 B) 80 C) 100 D) 110 E) 120

سبک ۲



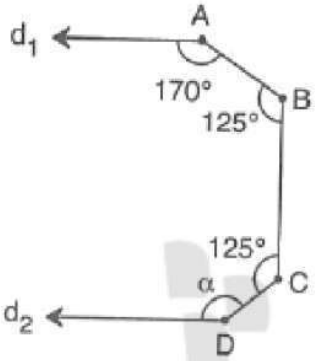
$d_1 // d_2$

$m(\widehat{ABC}) = 100°$

$m(\widehat{BCD}) = 140°$

$\Rightarrow m(\widehat{CDE}) = \alpha = ?$

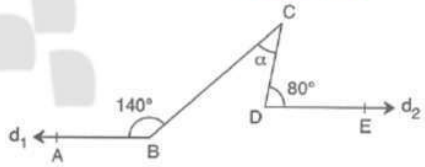
A) 100 B) 110 C) 120 D) 130 E) 140



$d_1 // d_2$

$\Rightarrow \alpha = ?$

سبک ۳



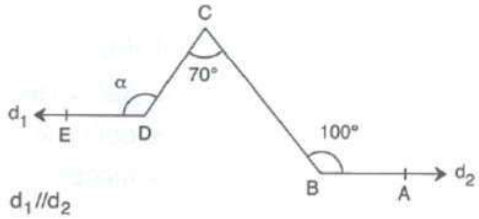
$d_1 // d_2$

$m(\widehat{ABC}) = 140°$

$m(\widehat{CDE}) = 80°$

$\Rightarrow m(\widehat{BCD}) = \alpha = ?$

A) 30 B) 40 C) 50 D) 60 E) 70



$d_1 // d_2$

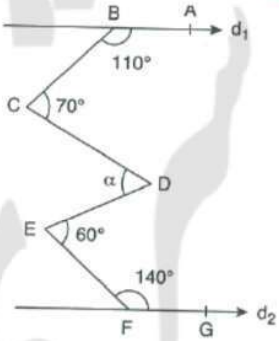
$m(\widehat{ABC}) = 100°$

$m(\widehat{BCD}) = 70°$

$\Rightarrow m(\widehat{CDE}) = \alpha = ?$

A) 150 B) 120 C) 110 D) 100 E) 90

سبک ۴



$d_1 // d_2$

$m(\widehat{ABC}) = 110°$

$m(\widehat{BCD}) = 70°$

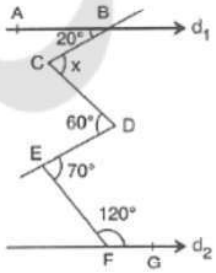
$m(\widehat{DEF}) = 60°$

$m(\widehat{EFG}) = 140°$

$m(\widehat{CDE}) = \alpha$

$\Rightarrow \alpha = ?$

A) 10 B) 20 C) 30 D) 40 E) 50



$d_1 // d_2$

$m(\widehat{ABC}) = 20°$

$m(\widehat{CDE}) = 60°$

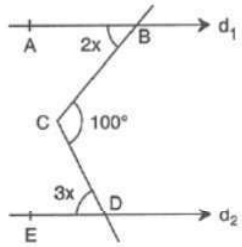
$m(\widehat{DEF}) = 70°$

$m(\widehat{EFG}) = 120°$

$\Rightarrow m(\widehat{BCD}) = x = ?$

A) 60 B) 70 C) 80 D) 100 E) 110

تمرین (سیگما بینی!)



$d_1 // d_2$

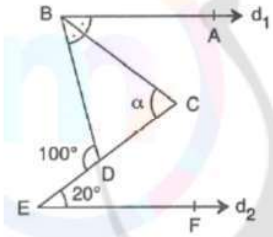
$m(\widehat{ABC}) = 2x$

$m(\widehat{CDE}) = 3x$

$m(\widehat{BCD}) = 100°$

$\Rightarrow x = ?$

A) 10 B) 15 C) 20 D) 25 E) 30



$d_1 // d_2$

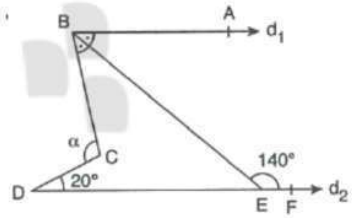
$m(\widehat{ABC}) = m(\widehat{CBD})$

$m(\widehat{BDE}) = 100°$

$m(\widehat{CEF}) = 20°$

$\Rightarrow m(\widehat{BCE}) = \alpha = ?$

A) 90 B) 80 C) 70 D) 60 E) 50



$d_1 // d_2$

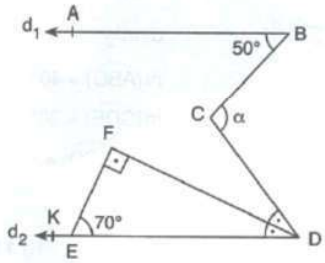
$m(\widehat{ABE}) = m(\widehat{CBE})$

$m(\widehat{CDF}) = 20°$

$m(\widehat{BEF}) = 140°$

$\Rightarrow m(\widehat{BCD}) = \alpha = ?$

A) 70 B) 80 C) 85 D) 90 E) 100



$d_1 // d_2$

$[EF] \perp [FD]$

$m(\widehat{ABC}) = 50°$

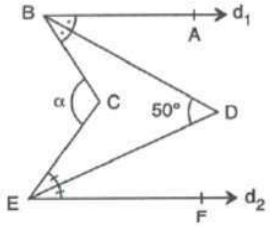
$m(\widehat{FED}) = 70°$

$m(\widehat{CDF}) = m(\widehat{FDK})$

$\Rightarrow m(\widehat{BCD}) = \alpha = ?$

A) 90 B) 80 C) 70 D) 60 E) 50

سبک ۵



$d_1 // d_2$

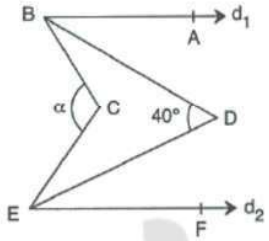
$m(\widehat{ABD}) = m(\widehat{CBD})$

$m(\widehat{CED}) = m(\widehat{DEF})$

$m(\widehat{BDE}) = 50°$

$\Rightarrow m(\widehat{BCE}) = \alpha = ?$

A) 25 B) 50 C) 75 D) 90 E) 100



$d_1 // d_2$

$m(\widehat{ABC}) = 2m(\widehat{CBD})$

$m(\widehat{CEF}) = 2m(\widehat{DEC})$

$m(\widehat{BDE}) = 40°$

$\Rightarrow m(\widehat{BCE}) = \alpha = ?$

A) 20 B) 40 C) 60 D) 80 E) 120

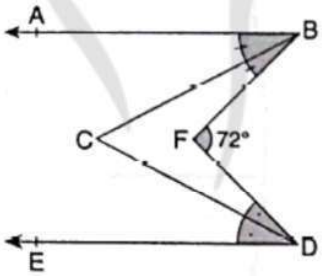
[BA // [DE

[BC] ve [DC] açıortaydır.

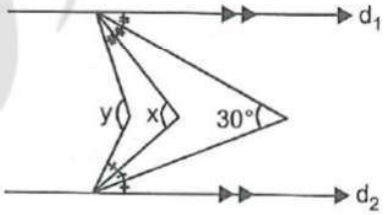
[BC] and [DC] are angle bisectors.

$m(\widehat{BFD}) = 72°$

$\Rightarrow m(\widehat{BCD}) = ?$

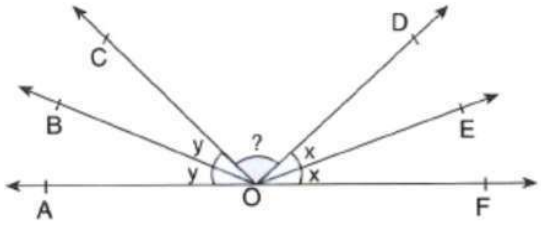


A) 36° B) 46° C) 56° D) 66° E) 76°



$d_1 // d_2 \Rightarrow y - x = ?$

A) 30 B) 35 C) 40 D) 45 E) 60



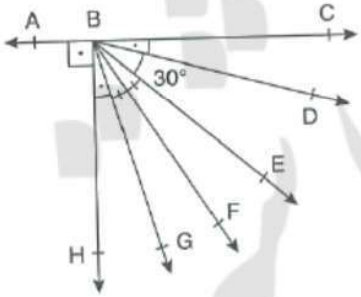
$m(\widehat{AOB}) = m(\widehat{BOC}) = y$,

$m(\widehat{DOE}) = m(\widehat{EOF}) = x$,

$m(\widehat{COE}) = 145°$,

$m(\widehat{BOD}) = 140° \Rightarrow$ **$m(\widehat{COD}) = ?$**

A) 110° B) 120° C) 125° D) 130° E) 140°



$AC \perp [BH$

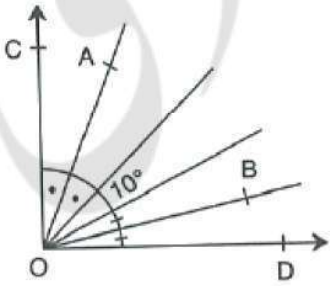
$m(\widehat{CBD}) = m(\widehat{HBG})$

$m(\widehat{GBF}) = m(\widehat{FBE})$

$m(\widehat{EBD}) = 30°$

$\Rightarrow m(\widehat{CBF}) = ?$

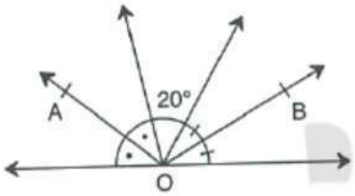
A) 15 B) 30 C) 45 D) 60 E) 75



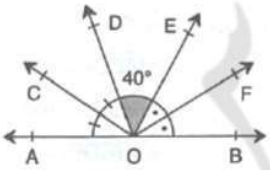
$[CO] \perp [OD]$

$\Rightarrow m(\widehat{AOB}) = ?$

تمرین



$\Rightarrow m(\widehat{AOB}) = ?$



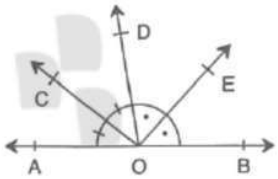
$m(\widehat{AOC}) = m(\widehat{COD})$

$m(\widehat{DOE}) = 40°$

$m(\widehat{EOF}) = m(\widehat{FOB})$

$\Rightarrow m(\widehat{COF}) = ?$

A) 50 B) 60 C) 100 D) 110 E) 130

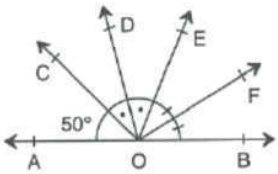


$m(\widehat{AOC}) = m(\widehat{COD})$

$m(\widehat{DOE}) = m(\widehat{EOB})$

$\Rightarrow m(\widehat{COE}) = ?$

A) 30 B) 60 C) 90 D) 120 E) 150



$m(\widehat{AOC}) = 50°$

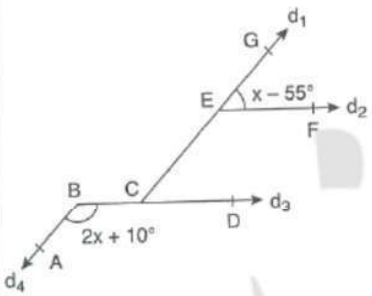
$m(\widehat{COD}) = m(\widehat{DOE})$

$m(\widehat{EOF}) = m(\widehat{FOD})$

$\Rightarrow m(\widehat{DOF}) = ?$

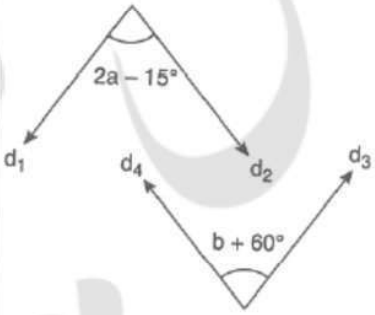
A) 50 B) 60 C) 65 D) 70 E) 90

تمرین

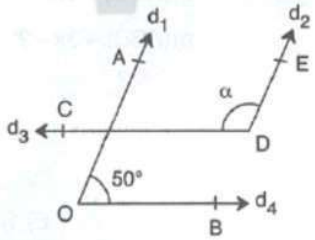


$d_1 // d_4$
$d_2 // d_3$
$m(\widehat{GEF}) = x - 55°$
$m(\widehat{ABC}) = 2x + 10°$
$\Rightarrow x = ?$

A) 15 B) 45 C) 60 D) 75 E) 85

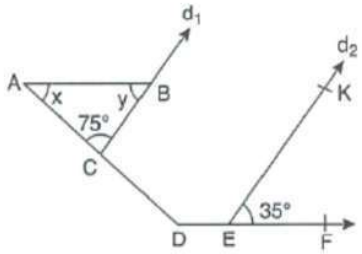


$d_1 // d_3$
$d_2 // d_4$
$a + b = 45°$
$\Rightarrow a - b = ?$



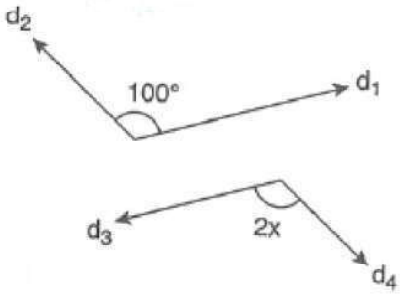
$d_1 // d_2$
$d_3 // d_4$
$m(\widehat{AOB}) = 50°$
$\Rightarrow m(\widehat{EDC}) = \alpha = ?$

A) 130 B) 100 C) 60 D) 50 E) 40

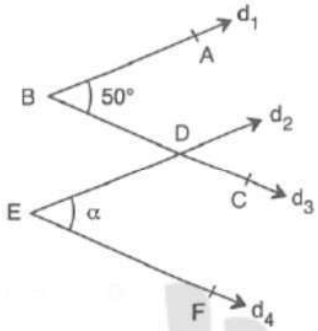


$d_1 // d_2$
$[AB] // [DF$
$m(\widehat{ACB}) = 75°$
$m(\widehat{KEF}) = 35°$
$m(\widehat{DAB}) = x$
$\Rightarrow m(\widehat{ABC}) = y = ?$

A) 25 B) 30 C) 35 D) 40 E) 45

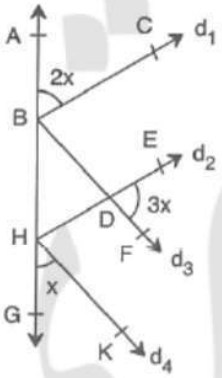


$d_1 // d_3$
$d_2 // d_4$
$\Rightarrow x = ?$



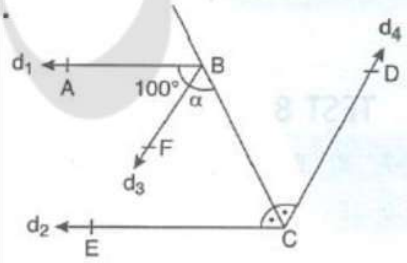
$d_1 // d_2$
$d_3 // d_4$
$m(\widehat{ABC}) = 50°$
$\Rightarrow m(\widehat{DEF}) = \alpha = ?$

A) 20 B) 30 C) 40 D) 50 E) 70



$d_1 // d_2$
$d_3 // d_4$
$m(\widehat{ABC}) = 2x$
$m(\widehat{EDF}) = 3x$
$\Rightarrow m(\widehat{GHK}) = x = ?$

A) 20 B) 30 C) 40 D) 50 E) 60



$d_1 // d_2$
$d_3 // d_4$
$m(\widehat{ABF}) = 100°$
$m(\widehat{ECB}) = m(\widehat{BCD})$
$\Rightarrow m(\widehat{FBC}) = \alpha = ?$

A) 40 B) 45 C) 50 D) 55 E) 60