

پاسخ سوالات یوس ۲۰۱۳

دانشگاه استانبول



Istanbul University

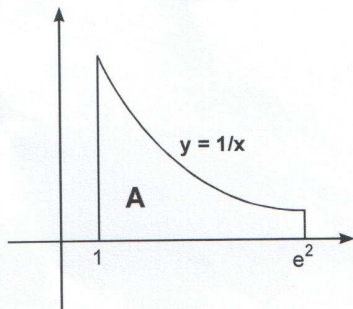
INTERNATIONAL STUDENTS' EXAM

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☎ ۰۲۱۹۱۳۰۵۹۰۵

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46.



$$y = \frac{1}{x}, x = 1, x = e^2 \Rightarrow A = ?$$

- A) 2 B) e C) 4 D) e^2 E) 8

$$A = \int_1^{e^2} \frac{1}{x} dx = [\ln x]_1^{e^2} = \ln e^2 - \ln 1 = 2$$

(A)

47.

$$\begin{bmatrix} x & 2 \\ y & -1 \end{bmatrix}^2 = \begin{bmatrix} 6 & 2 \\ 1 & 3 \end{bmatrix} \Rightarrow (x, y) = ?$$

- A) (1, 2) B) (1, 0) C) (2, 0)
D) (2, 1) E) (2, 3)

$$\begin{bmatrix} x & 2 \\ y & -1 \end{bmatrix} \begin{bmatrix} x & 2 \\ y & -1 \end{bmatrix} = \begin{bmatrix} x^2 + 2y & 2x - 2 \\ xy - y & 2y + 1 \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 1 & 3 \end{bmatrix}$$

$$\begin{cases} 2x - 2 = 2 \\ 2y + 1 = 3 \end{cases} \Rightarrow (x, y) = (2, 1)$$

(D)

48.

$$|x| < 1 \Rightarrow \sum_{n=1}^{\infty} x^n = ?$$

- A) $\frac{1}{1-x}$ B) $\frac{1}{1+x}$ C) $\frac{x}{1-x}$
D) 1 E) 10

$$\sum_{n=1}^{\infty} x^{n-1} = \frac{1}{1-x} \Rightarrow \text{سری هندسی}$$

A

$$\sum_{n=1}^{\infty} x^n = x \sum_{n=1}^{\infty} x^{n-1} = x \left(\frac{1}{1-x} \right) = \frac{x}{1-x}$$

(C)

49.

$$\tan(x + 45^\circ) = 2$$

$$\sin x = ?$$

- A) $\frac{1}{\sqrt{10}}$ B) $\frac{1}{3}$ C) $\frac{1}{2}$
D) $\frac{2}{\sqrt{10}}$ E) $\frac{3}{\sqrt{10}}$

$$\tan(x + 45^\circ) = \frac{\tan x + \tan 45^\circ}{1 - \tan x \tan 45^\circ} = \frac{\tan x + 1}{1 - \tan x} = 2$$

$$\Rightarrow \tan x = \frac{1}{3} \Rightarrow \frac{\sin x}{\cos x} = \frac{1}{3} \Rightarrow \cos x = 3 \sin x$$

$$\sin^2 x + \cos^2 x = 1 \Rightarrow \sin^2 x + 9 \sin^2 x = 1 \Rightarrow 10 \sin^2 x = 1 \Rightarrow \sin x = \frac{1}{\sqrt{10}}$$

(A)

50.

$$\log_2 3 = x$$

$$\log_{12} 24 = ?$$

- A) $\frac{5}{x+2}$ B) $\frac{x+3}{x+2}$ C) $\frac{3x+1}{x+2}$
D) $\frac{2x+3}{x+3}$ E) $\frac{2x+1}{x+2}$

$$\log_{12} 24 = \frac{\log_2 24}{\log_2 12}$$

$$\Rightarrow \frac{\log_2 3 + \log_2 8}{\log_2 4 + \log_2 3} = \frac{x + 3}{2 + 2} = \frac{x+3}{x+2}$$

(B)

51.

$$\log_4 x = 3$$

$$\log_2 x = y$$

$$3x^3 y = ?$$

- A) $\frac{81}{2}$ B) 24 C) 12 D) 8 E) 6

$$\log_4 x = 3 \Rightarrow x^3 = 4 \Rightarrow \frac{3y}{2} = 4 \Rightarrow 3y = 8 \Rightarrow y = \frac{8}{3}$$

$$3x^3 y = 3 \times 4 \times \frac{8}{3} = 32$$

(D)

21

52.

$$\lim_{x \rightarrow 3} f(x) = 4$$

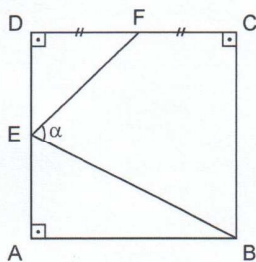
$$\lim_{x \rightarrow 3} \frac{(f(x) - 3)(f^2(x) - 4f(x) + 1)}{x + 3} = ?$$

- A) 0 B) $\frac{1}{6}$ C) $\frac{1}{3}$ D) $\frac{1}{2}$ E) 5

$$\lim_{x \rightarrow 3} \frac{(4-3)(16-4 \cdot 4+1)}{6} = \frac{1}{6}$$

(B)

53.



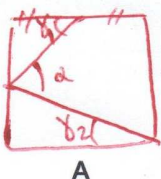
$$\widehat{m(\angle BEF)} = \alpha \quad |DF| = |FC|,$$

$$|AB| = |BC| = |CD| = |DA|$$

$$\frac{|DE|}{|EA|} = 2$$

$$\tan \alpha = ?$$

- A) $-\frac{1}{3}$ B) $\frac{1}{3}$ C) $\frac{2}{3}$ D) 2 E) 3



A

$$\delta_1 + \delta_2 = \alpha \Rightarrow \tan \delta_1 = \frac{DE}{DF} = \frac{2/3}{1/2} = \frac{4}{3}$$

$$\Rightarrow \tan \delta_2 = \frac{EA}{AB} = \frac{1}{3}$$

$$\tan \alpha = \tan(\delta_1 + \delta_2) = \frac{\tan \delta_1 + \tan \delta_2}{1 - \tan \delta_1 \tan \delta_2} = \frac{4/3 + 1/3}{1 - 4/9} = \frac{5/3}{5/9} = 3$$

(E)

54.

$$x^2 + (m^2 - 16)x - 2 + m = 0$$

$$x_1 + x_2 = 0$$

$$m = ?$$

- A) -4 B) -2 C) 0 D) 2 E) 4

$$\begin{cases} (x_1 + x_1)(x_1 + x_2) = x^2 + (x_1 + x_2)x + x_1 x_2 \\ x_1 + x_2 = 0 \rightarrow x_1 = -x_2 \Rightarrow m^2 - 16 = 0 \\ m^2 = 16 \end{cases}$$

$$m = \pm 4$$

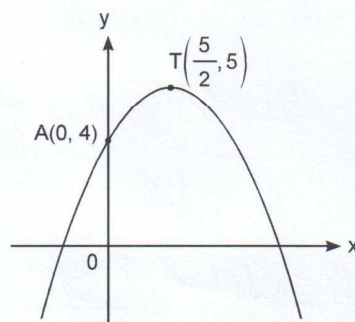
$$x^2 - 2 + m = 0$$

$$x^2 = m + 2 \rightarrow -m + 2 = 0 \Rightarrow m = 2 \Rightarrow m = 4$$

می تواند صفرم باشد چون دو جواب دارد.

(A)

55.



$$y = ax^2 + bx + c$$

$$b = ?$$

- A) $-\frac{4}{25}$ B) $-\frac{4}{5}$ C) $\frac{4}{25}$ D) $\frac{4}{5}$ E) $\frac{5}{4}$

$$A \Rightarrow x = 0, y = 4 \Rightarrow 4 = c$$

$$y' = 2ax + b \Rightarrow y'(\frac{5}{2}) = 0 \Rightarrow \frac{5a}{2} + b = 0 \Rightarrow a = -\frac{b}{5}$$

$$y(\frac{5}{2}) = 5 \Rightarrow \frac{-b}{5} \times (\frac{5}{2})^2 + b \times \frac{5}{2} + 4 = 5$$

$$-\frac{5b}{4} + \frac{5b}{2} + 4 = 5 \Rightarrow$$

$$\frac{5b}{4} = 1 \Rightarrow b = \frac{4}{5}$$

(D)

56. $A = \begin{bmatrix} 1996 & 1998 \\ 1997 & 1999 \end{bmatrix}_{2 \times 2}$

$\det(A) = ?$

- A) 1996 B) 0 C) -1 D) -2 E) -1996

$$\det(A) = (1996 \times 1999) - (1997 \times 1998)$$

$$1996 = 2000 - 4 \rightarrow 2000 = a$$

$$\Rightarrow \det(A) = (a-4)(a-1) - (a-3)(a-2)$$

$$(a^2 - 5a + 4) - (a^2 - 5a + 6) = -2$$

(D)

57. $f(x^2 + 2x + 4) = 3x^2 + 6x + 7$, $f^{-1}(4) = ?$

- A) -5 B) $-\frac{5}{3}$ C) -1 D) 3 E) 4

$$x^2 + 2x + 4 = f^{-1}(3x^2 + 6x + 7)$$

$$3x^2 + 6x + 7 = 4 \Rightarrow 3x^2 + 6x + 3 = 0$$

$$x^2 + 2x + 1 = 0 \Rightarrow (x+1)^2 = 0 \rightarrow x = -1$$

$$f^{-1}(4) = (-1)^2 + 2(-1) + 4 = 3$$

(D)

58. $A(1, 3)$, $B(-2, 4)$, $C(a, 2)$, $D(-1, 0)$

$$\langle \overrightarrow{AB}, \overrightarrow{DC} \rangle = 26, a = ?$$

- A) -9 B) -8 C) 0 D) 8 E) 9

$$\overrightarrow{AB} = (-2-1, 4-3) = (-3, 1)$$

$$\overrightarrow{DC} = (-1+a, 2)$$

$$\langle \overrightarrow{AB}, \overrightarrow{DC} \rangle = -3(a+1) + 2 =$$

$$-3a - 3 + 2 = 26$$

$$a = -9$$

(A)

59. $\sqrt[3]{2\sqrt{21-\sqrt{23+\sqrt{4}}}} = ?$

- A) 6 B) 5 C) 4 D) 3 E) 2

$$\sqrt[3]{2\sqrt{21-\sqrt{23+\sqrt{4}}}}$$

$$\sqrt[3]{2\sqrt{21-\sqrt{23+2}}}$$

$$\sqrt[3]{2\sqrt{21-\sqrt{25}}}$$

$$\sqrt[3]{2\sqrt{21-5}}$$

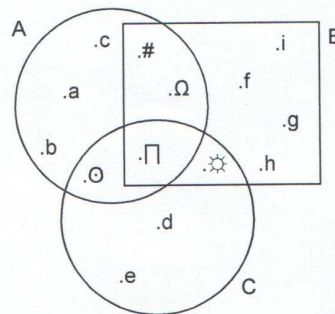
$$\sqrt[3]{2\sqrt{16}}$$

$$\sqrt[3]{2 \times 4}$$

$$\sqrt[3]{8} = 2$$

(E)

60.



$$[(A \cup B) \setminus C'] = ?$$

- A) \emptyset B) $\{a, b, c\}$ C) $\{\#, \Omega\}$
D) $\{o, \Pi, \odot\}$ E) $\{f, g, h, i\}$

$$(A \cup B) \setminus C' = (A \cup B) - C' = (A \cup B) \cap C =$$

$$= \{o, \Pi, \odot\}$$

(D)

61.

$$\begin{array}{r|l} P(x) & x+1 \\ \hline & A(x) \\ \hline - & -6 \end{array} \quad \begin{array}{r|l} P(x) & x-2 \\ \hline & B(x) \\ \hline - & 6 \end{array}$$

$$\begin{array}{r|l} P(x) & (x+1)(x-2) \\ \hline & C(x) \\ \hline - & ax+b \end{array} \quad ax+b=?$$

- A) $4x+2$ B) $4x-2$ C) $2x-2$
D) $x-4$ E) $2x+4$

$$P(-1) = -6 \quad P(2) = 6$$

$$P(x) = C(x+1)(x-2) + ax + b$$

$$\Rightarrow P(1) = -a + b = -6 \Rightarrow 3a = 12$$

$$P(2) = 2a + b = 6 \Rightarrow \begin{cases} a = 4 \\ b = -2 \end{cases}$$

(B)

62. $f(x) = 3^{2x+1}$

$$f(2x) = m \cdot (f(x))^2$$

$$m = ?$$

- A) 3 B) 2 C) 1 D) $\frac{1}{3}$ E) $\frac{1}{9}$

$$f(x) = 3^{2x+1} \Rightarrow f(2x) = 3^{4x+1}$$

$$[f(x)]^2 = 3^{4x+2}$$

A

$$[f(x)]^2 = (3^{4x+1}) \times 3 = f(2x) \times 3$$

$$f(2x) = \frac{1}{3} [f(x)]^2$$

(D)

63.

$$\frac{x^2+5x+6}{x^3+x^2-2x} \cdot \frac{x^3-x}{x^2+6x+5} = ?$$

- A) $\frac{x+5}{x-1}$ B) $\frac{x-3}{x+1}$ C) $\frac{x-2}{x+2}$
D) $\frac{x+1}{x+3}$ E) $\frac{x+3}{x+5}$

$$\frac{(x+2)(x+3)}{x(x+2)(x-1)} \times \frac{x(x+1)(x-1)}{(x+5)(x+1)} = \frac{x+3}{x+5}$$

(E)

64.

$$i^2 = -1$$

$$\sqrt{-2} \cdot \sqrt{-5} \cdot \sqrt{-10} = ?$$

- A) $-2i$ B) $-5i$ C) $-10i$
D) $10i$ E) 10

$$\sqrt{2}i \times \sqrt{5}i \times \sqrt{10}i = \sqrt{100}i^3 = -10i$$

(C)

24

65. $f(x) = (\cos 3x)^3$

$f'\left(\frac{\pi}{3}\right) = ?$

- A) -1 B) 0 C) 1 D) 2 E) 3

$$f(x) = (\cos 3x)^3 \Rightarrow f'(x) = 3(\cos 3x)^2 (\cos 3x)'$$

$$= 3(\cos 3x)^2 \times (-3 \sin 3x) = -9 \cos^2 3x \sin 3x$$

$$= 0$$

(B)

66. $f(x) = e^{2\sin x}$

$f'(x) = ?$

- A) $2\cos x e^{2\sin x}$ B) $2\sin x e^{2\sin x}$
C) e^{2x-1} D) e^{x+2}
E) $e^{2\sin x}$

$$f(x) = e^{2\sin x}$$

$$f'(x) = (2\sin x)' e^{2\sin x} = 2\cos x e^{2\sin x}$$

(A)

A

67. $\int_1^3 \frac{1}{x \ln 2} dx = ?$

- A) $\frac{\ln 3}{\ln 2}$ B) $2 \ln 2$ C) $\frac{1}{\ln 2}$
D) $\ln 4$ E) e^{2x}

$$\int_1^3 \frac{1}{x \ln 2} dx = \frac{1}{\ln 2} (\ln x) \Big|_1^3 = \frac{\ln 3}{\ln 2}$$

(A)



68. $\frac{2}{5} < x < \frac{3}{4}$

$x = ?$

- A) $\frac{3}{10}$ B) $\frac{1}{3}$ C) $\frac{1}{2}$
D) $\frac{4}{5}$ E) $\frac{5}{6}$

$$x > \frac{2}{5} \Rightarrow \left\{ \frac{1}{2}, \frac{4}{5}, \frac{5}{6} \right\}$$

$$x < \frac{3}{4} \Rightarrow \left\{ \frac{3}{10}, \frac{1}{3}, \frac{1}{2} \right\}$$

$\Rightarrow \frac{1}{2}$ (C)

69. $x < \frac{5}{3}$

$|5-3x| - \sqrt{9x^2 - 30x + 25} + 3x - 1 = ?$

- A) 4 B) $3x - 1$ C) $9 - 3x$
 D) $3x - 9$ E) $1 - 3x$

$|5-3x| - \sqrt{(3x-5)^2} - 1$

$|5-3x| - |3x-5| - 1$

$3x - 1$

(B)

70. $x = 3 + 2^{a-1}$

$y = 5 - 2^{a+1}$

$\frac{x}{y} = ?$

- A) $\frac{y-7}{4y}$ B) $\frac{17+y}{4y}$ C) $\frac{17-y}{4y}$

- D) $\frac{y+7}{4y}$ E) $\frac{1}{4} - \frac{7}{y}$

$x = 3 + 2^{a-1} \Rightarrow -x = -3 - 2^{a-1}$

$-4x = -12 - 2^{a+1}$

$\Rightarrow -4x + 17 = 5 - 2^{a+1}$

A

$-4x + 17 = y \Rightarrow x = \frac{17-y}{4}$

$\frac{x}{y} = \frac{17-y}{4y}$

(C)

71. $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{1}{5}$

$2a - 3c + 5e = -8$

$2b + 5f = 35$

$d = ?$

- A) -25 B) -5 C) 1 D) 5 E) 25

$\frac{2a-3c+5e}{2b-3d+5f} = \frac{1}{5}$

$\frac{-8}{35-3d} = \frac{1}{5} \Rightarrow 35-3d = -40$

$3d = 75 \Rightarrow d = 25$

(E)

72. $\frac{3}{4} + \left(\frac{2}{5} - \frac{1}{2}\right) - \left(\frac{3}{2} + \frac{1}{4}\right) = ?$

- A) $-1\frac{1}{5}$ B) $-1\frac{1}{10}$ C) $1\frac{1}{10}$

- D) $1\frac{1}{5}$ E) $1\frac{1}{4}$

$\frac{3}{4} + \left(\frac{2}{5} - \frac{1}{2}\right) - \left(\frac{3}{2} + \frac{1}{4}\right)$

$\frac{3}{4} + \frac{-1}{10} - \frac{7}{4} = \frac{-4}{4} - \frac{1}{10} = -1\frac{1}{10}$

(B)

73. $\left(2 + \frac{3}{8}\right) \div \left(\frac{2}{3} - \frac{1}{6}\right) = ?$

A) $4\frac{3}{4}$

B) $4\frac{1}{4}$

C) $3\frac{1}{3}$

D) $3\frac{1}{4}$

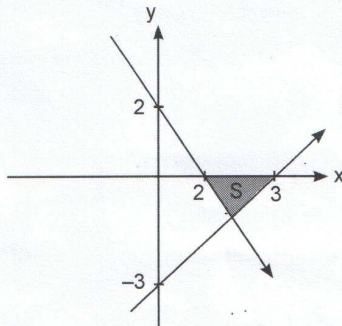
E) $2\frac{1}{3}$

$\left(2 + \frac{3}{8}\right) \div \left(\frac{2}{3} - \frac{1}{6}\right)$

$\frac{19}{8} \div \frac{3}{8} = \frac{19}{8} \times \frac{8}{3} = \frac{19}{3}$

$\frac{19}{3} = 6\frac{1}{3}$ (A)

74.



S = ?

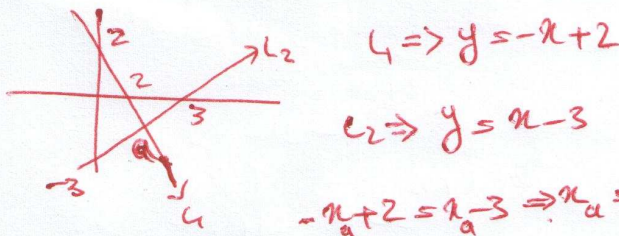
A) $\frac{1}{4}$

B) $\frac{1}{2}$

C) $\frac{5}{4}$

D) 2

E) 3

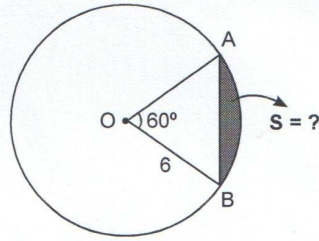


A

$S = \frac{1}{2} \times \frac{5}{2} \times \frac{1}{2} = \frac{1}{4}$

(A)

75.



A) 2π

B) $2\pi - 9\sqrt{3}$

C) $4\pi + 9\sqrt{3}$

D) $6\pi - 9\sqrt{3}$

E) $6\pi - 9$

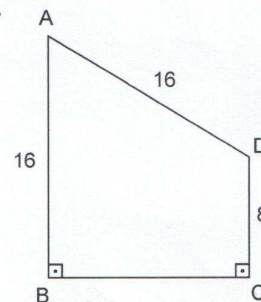
$S = \pi \times 36 \times \frac{60}{360} = 6\pi$

$S = 36\pi \times \frac{1}{6} = 6\pi$

$S = 6 \times \frac{\sqrt{3}}{4} \times 6 = 9\sqrt{3}$

$S = 6\pi - 9\sqrt{3}$ (D)

76.



$|DC| \perp |BC|$

$|AB| \perp |BC|$

$|AB| \perp |AD| = 16$

$|DC| = 8$

$m(\widehat{BAD}) = ?$

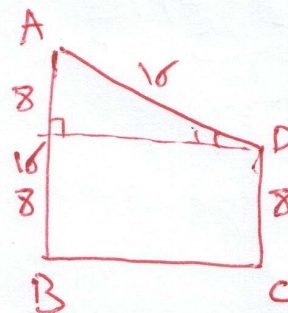
A) 30°

B) 40°

C) 50°

D) 60°

E) 70°



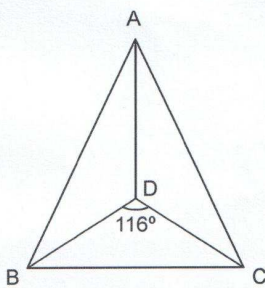
$\sin D_1 = \frac{8}{16} = \frac{1}{2} \Rightarrow D_1 = 30^\circ$

$\widehat{D}_1 + 90^\circ + \widehat{A} = 180^\circ$

$\widehat{A} = 60^\circ$

(D)

77.



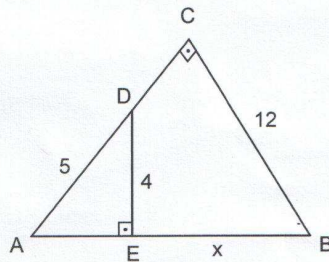
$$\begin{aligned} m(\widehat{DBC}) &= m(\widehat{ABD}) \\ m(\widehat{BCD}) &= m(\widehat{DCA}) \\ m(\widehat{BDC}) &= 116^\circ \\ m(\widehat{DAC}) &= ? \end{aligned}$$

- A) 25 B) 26 C) 50 D) 52 E) 64

حل کنیم بیاریم نقطه است
بنابراین $A_1 = A_2$

$$\begin{aligned} \triangle ABC &= 2A_1 + 2B_1 + 2C_1 = 180 \\ \triangle BDC &= B_1 + C_1 + 116 = 180 \Rightarrow B_1 + C_1 = 64 \\ 2A_1 + 2 \times 64 &= 180 \Rightarrow 2A_1 = 52 \quad A_1 = 26 \quad \text{(B)} \end{aligned}$$

78.



$$\widehat{AED} = \widehat{BCA} = 90^\circ$$

$$\begin{aligned} |AD| &= 5 \\ |BC| &= 12 \\ |DE| &= 4 \\ |EB| &= x \end{aligned}$$

x = ?

- A) 12 B) 13 C) 15 D) 18 E) 19

$\Rightarrow \triangle ACB \sim \triangle AED$

$$\frac{AD}{AB} = \frac{DE}{BC} = \frac{1}{3}$$

$$\Rightarrow AB = 15$$

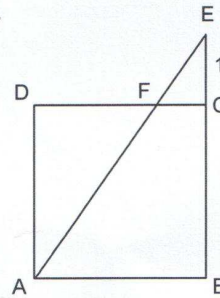
A

$$x = 15 - 3 = 12$$

(A)

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79.

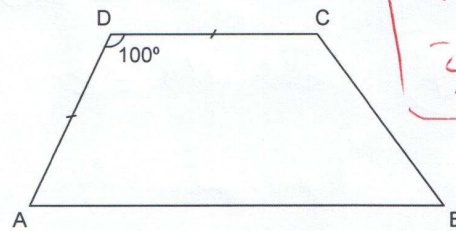


$$\begin{aligned} |AB| &= |BC| = |CD| = |AD| \\ |EC| &= 1 \text{ cm} \\ |AE| &= 5 \text{ cm} \\ |DC| &= ? \end{aligned}$$

- A) 2 B) 3 C) 4 D) 5 E) 6

$AE = 5 \Rightarrow AC < 5$
 $2a^2 \in AC^2 \Rightarrow 2a^2 < 25 \Rightarrow a < \frac{5}{\sqrt{2}}$
 $a^2 + (a+1)^2 = 5^2 \Rightarrow a^2 + a^2 + 2a + 1 = 25$
 $2a^2 + 2a - 24 = 0 \Rightarrow a^2 + a - 12 = 0$
 $(a+4)(a-3) = 0 \Rightarrow a = 3$
با هم ظاهر دانستن زاویه
۳ و ۴ و ۵
سایر سی به حل سی

80.

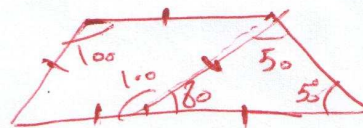


$$|AB| \parallel |DC|, |AD| = |DC| = \frac{|AB|}{2}$$

$$m(\widehat{ADC}) = 100^\circ$$

$$m(\widehat{DCB}) = ?$$

- A) 130° B) 100° C) 80° D) 50° E) 30°



TEST BİTTİ.

CEVAPLARINIZI KONTROL EDİNİZ.

END OF TEST.

PLEASE CHECK YOUR ANSWERS.

(D)